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## Testing Scenarios of Lorentz Symmetry Violation Generated at the Planck Scale

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### Abstract

Using new theoretical tools, which allow to better understand ultra-high energy (UHE) dynamics, several patterns of Lorentz symmetry violation (LSV) are studied and compared with experiment. It is claimed that quadratically deformed relativistic kinematics (QDRK), where the parameter driving LSV varies like the square of the energy scale, remains the best suited pattern to describe LSV generated at the Planck scale. Implications of existing data are discussed and prospects are presented having in mind next-generation experiments.

### 1. Deformed Relativistic Kinematics

We have proposed [4-7] several LSV patterns in the vacuum rest frame (VRF) producing deformed Lorentz symmetries (DLS) and deformed relativistic kinematics (DRK). In particular, QDRK [7-16] is characterized by:

$$E = (2\pi)^{-1} h c a^{-1} e(k a) \quad (1)$$

$E$  being the energy,  $h$  the Planck constant,  $a$  the fundamental length,  $c$  the speed of light,  $k$  the wave vector and  $[e(k a)]^2$  a convex function of  $(k a)^2$ . For  $k a \ll 1$ :

$$e(k a) \simeq [(k a)^2 - \alpha (k a)^4 + (2\pi a)^2 h^{-2} m^2 c^2]^{1/2} \quad (2)$$

$\alpha$  being a model-dependent constant that may be in the range  $0.1 - 0.01$  for full-strength violation of Lorentz symmetry at the fundamental length scale, and  $m$  the mass of the particle. For momentum  $p \gg mc$ , we get:

$$E \simeq p c + m^2 c^3 (2 p)^{-1} - p c \alpha (k a)^2 / 2 \quad (3)$$

LDRK (linearly deformed relativistic kinematics) is another fashionable model (see [1,2] and [14,15,17,18]) and corresponds to:

$$e(k a) \simeq [(k a)^2 - \beta (k a)^3 + (2\pi a)^2 h^{-2} m^2 c^2]^{1/2} \quad (4)$$

$\beta$  being a model-dependent constant. For  $p \gg mc$ :

$$E \simeq p c + m^2 c^3 (2 p)^{-1} - p c \beta (k a) / 2 \quad (5)$$

Consistency requires that, for large bodies,  $\alpha \propto m^{-2}$  and  $\beta \propto m^{-1}$ . More generally, DRK models can be built where for  $k a \ll 1$  and  $p \gg mc$  the deformation term in the expression of  $E$  varies like  $-\kappa p (k a)^\delta$ ,  $\delta$  being an arbitrary positive real number and  $\kappa$  a constant. Then, consistency requires  $\kappa \propto m^{-\delta}$  for large bodies. Other models can be generated from the mixing of superbradyons with "ordinary" particles (see [14] and references therein, as well as updated discussions in [17,18]). In another paper submitted to this conference, we discuss some potentialities of superdradion mixing in DRK phenomenology.

As shown in [17,18], both QDRK and LDRK can be obtained from formal Lorentz symmetric patterns using suitable transformations. The approach followed there is an extension of that proposed by Magueijo and Smolin [21,22] within the Doubly Special Relativity scheme (see, f.i., [1,2] and references therein). The pattern of [17,18] allows to make the transformation operators formally unitary and to use extra dimensions in the dynamical description of 4-dimensional DRK. It also makes possible to build extensions of the Kirzhnits-Chechin (KCh) model [19,20] generalizing its Finsler space approach and setting a natural bridge with QDRK. Such as initially formulated, the KCh model was unable to really account for the absence of the Greisen-Zatsepin-Kuzmin (GZK) cutoff and, although it satisfied the requirement  $\alpha \propto m^{-2}$  for large bodies when set in a form close to QDRK, it presented the same feature for elementary particles which is an obvious drawback [18]. Our approach allows to build new models similar in their formal structure to the KCh model but able to reproduce the QDRK pattern.

A characteristic feature of DRK models is that, above some critical energy scale and even for a very small LSV, the properties of the ultra-high energy particles (UHEPs) drastically change. This is true not only for their apparent interaction properties, but also for their internal structure. In previous papers [14,15], we had already pointed out that the interaction properties of a ultra-high energy cosmic ray (UHECR) particle in the LDRK pattern were expected to be modified too early, as energy increases, for consistent phenomenology. The same conclusion can be reached by looking at the internal structure of the UHEP using the operator formalism [17,18]. The basic mechanism can be illustrated from a simple soliton model developed in [7-10]. Assume that, in the VRF, a wave function in a relativistic dynamical model is perturbed by nonlocal corrections with distance scale  $a$ , so that the initial Lorentz symmetry is violated and we get:

$$\phi(x \pm a) - \phi(x) = \sum_{n=0}^{\infty} \phi^{(n)}(x) (\pm a)^n (n!)^{-1} \quad (6)$$

where  $\phi^{(n)} = d^n \phi / dx^n$ ,  $\phi^0$  is the initial solution with Lorentz symmetry and  $\phi(x)$  is the fixed-time wave function. Then, if the model has solitons, the natural dimensionless parameter to describe LSV and to be compared with  $\gamma_R^{-2}$  ( $\gamma_R$  is the unperturbed relativistic Lorentz factor), will be  $\xi_2 = \alpha (a \gamma_R)^2 \Delta^{-2}$  for QDRK and  $\xi_1 = \beta a \gamma_R \Delta^{-1}$  for LDRK, where  $\Delta$  is the characteristic size scale of the

unperturbed relativistic soliton at rest [9,10,17,18]. When  $\xi_1$  ( $\xi_2$ ) becomes larger than  $\gamma_R^{-2}$ , the effective size scale of the UHE soliton changes. It can be readily checked that the change in size of the soliton induced by LSV occurs at lower energies in the case of LDRK, leading to phenomenological problems [17,18].

## 2. QDRK

The deformation approximated by  $\Delta E = - p c \alpha (k a)^2/2$  in the right-hand side of (3) has important consequences for UHECR. At energies above  $E_{trans} \approx \pi^{-1/2} h^{1/2} (2 \alpha)^{-1/4} a^{-1/2} m^{1/2} c^{3/2}$ , the deformation  $\Delta E$  dominates over the mass term  $m^2 c^3 (2 p)^{-1}$  and modifies all kinematical balances. For a proton, taking  $a$  to be the Planck length, a typical scale for  $E_{trans}$  is  $\sim 10^{19} \text{ eV}$ . Furthermore, there is a sharp fall of partial and total cross-sections for cosmic-ray energies above  $E_{lim} \approx (2 \pi)^{-2/3} (E_T a^{-2} \alpha^{-1} h^2 c^2)^{1/3}$ , where  $E_T$  is the target energy. For microwave background photons and  $\alpha \approx 0.1$ ,  $E_{lim} \approx 10^{19} \text{ eV}$ . Such figures naturally allow to explain the absence of GZK cutoff. The change in internal structure of the proton is expected to occur at a similar energy scale. This is compatible with experimental data and leads to interesting predictions that can be checked by future experiments [14-18]. Using the operator formalism and going to the rest frame of the UHECR, it is possible to monitor the evolution of its internal structure, not only for the standard QDRK model but also for more general patterns. Suitable techniques can be to go to the formally symmetric reference system, perform the Lorentz transformation and go back to the physical reference system, or to directly write the deformation as a spontaneous LSV in the physical reference system. The analysis of the internal structure of a UHEP in its rest frame suggests in turn more precise tests requiring a careful study of the first interactions of UHECR in the atmosphere. This will be, if feasible, a difficult experimental task but it may be possible with next generation experiments [17,18].

## 3. LDRK

LDRK was discarded in our 1997 and subsequent DRK papers, but is often proposed (see [1,2] and references therein) for cosmic-ray,  $TeV$  gamma-ray and gravitational-wave phenomenology. The deformation yields effects at comparatively low energies, with  $E_{trans} \approx \pi^{-1/3} h^{1/3} (2 \beta)^{-1/3} a^{-1/3} m^{2/3} c^{5/3}$  and  $E_{lim} \approx (2 \pi)^{-1/2} (E_T a^{-1} \beta^{-1} h c)^{1/2}$ . The  $E_{trans}$  scale, for a UHE proton, is now  $\sim 10^{16} \text{ eV}$  and similarly for the change of its internal structure. For atmospheric targets,  $E_{lim}$  falls naturally in the  $\sim 10^{19} \text{ eV}$  region if  $a$  is the Planck length. No existing phenomenological analysis or experimental data tend to confirm such predictions. Although LDRK is often used to explain possible anomalies of data at comparatively low energy ( $TeV$  scale), the basic difficulties generated by its global properties appear too hard to overcome [14,15,17,18].

#### 4. Conclusion

QDRK remains the best suited model for LSV in UHCR phenomenology. Requiring simultaneously the absence of GZK cutoff in the region  $E \approx 10^{20} \text{ eV}$ , and that cosmic rays with  $E$  below  $\approx 3 \cdot 10^{20} \text{ eV}$  deposit most of their energy in the atmosphere, leads [11,14] to:  $10^{-72} \text{ cm}^2 < \alpha a^2 < 10^{-61} \text{ cm}^2$ , equivalent to  $10^{-20} < \alpha < 10^{-9}$  for  $a \approx 10^{-26} \text{ cm}$  ( $\approx 10^{21} \text{ GeV}$  scale). Assuming full-strength LSV forces  $a$  to be in the range  $10^{-36} \text{ cm} < a < 10^{-30} \text{ cm}$ . Thus, the simplest version of QDRK naturally fits with the expected potential role of Planck-scale dynamics. The same considerations apply to the transition in the internal structure of a UHE proton. Above  $E_{trans}$ , the particle looks closer to a Planck-scale object than to the conventional particles of current literature. Even a small LSV at the Planck scale can be at the origin of dramatic changes in the structure and interaction properties of particles at much lower energy scales (above  $E_{trans}$ ). More details, using explicitly the operator formalism, can be found in references [17,18] as well as in subsequent papers of the same series (*Deformed Lorentz symmetry and High-Energy Astrophysics*, see arXiv.org ).

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